Journal of Statistics and Computer Science
Vol. 1, Issue 2, 2022, pp. 147-158
https://doi.org/10.47509/JSCS.2022.v01i02.04

Estimation of Stress Strength Reliability for New Generalized Pareto Distribution using Advanced Sampling Methods

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Abstract: In reliability theory, the issue with stress strength estimation has many applications. The $R = P(X < Y)$ stress-strength reliability is applied to describe the survival rate of a component which has X as stress applied and Y as the strength of the system. In the present article, we investigate the R estimation in case where two independent random variables X and Y are used each having New Generalized Pareto distribution based on some advanced sampling methods. The suggested sampling methods are "simple random sampling, ranked set sampling and percentile ranked set sampling". We propose the maximum likelihood estimator of R, when observations for two random variables are chosen using different sampling approaches. Output of proposed estimators for R based on percentile ranked set sampling and ranked set sampling with the simple random sampling equivalent is compared through the simulation study. Estimator efficiencies are measured using ratio of mean square errors. For illustrations, real data set is also analyzed.
Keywords: Maximum Likelihood, New generalized Pareto distribution, Percentile ranked set sampling, Ranked set sampling, Stress strength reliability.

MSC 2020 subject classification: 62D99, 62F99.

1 Introduction

The system reliability has wide range of uses in many areas like in reliability analysis, medicine, quality control and engineering. The equation $R = P(X < Y)$ defines the stress strength reliability, with X as the strength and Y is the stress applied against strength. Obviously a system will fail in case the applied stress is more than the component strength. Concept of stress strength model was introduced by Birnbaum (1956). It was Church and Harris (1970) who first introduced the term stress strength. An extensive study on the model of stress strength was carried out by Johnson (1988). An exhaustive study of different stress strength distributions till 2001 can be traced in the work of Kotz and Pensky (2003). Krishnamoorthy, Mukherjee, and Guo (2007) considered exponential strength and stress random variables. Kundu and Gupta (2005) and Raqab, Madi, and Kundu (2008) considered the R estimation for two and three parameter generalized exponential distributions. Al-Mutairi, Ghitany, and Kundu (2013), Ghitany, Al-Mutairi, and Aboukhamseen (2015) and Rezaei, Sharafi, Behboodian, and Zamani (2018) considered same problem in case of Lindley, power Lindley and generalized Lindley type 5. Al-Omari, Almanjahie, Hassan, and Nagy (2020) considered the stress strength reliability estimation for exponentiated Pareto distribution. Recently Saini, Chaturvedi, and Garg (2021) considered stress strength reliability estimation for generalized Maxwell failure distribution under progressive first failure censoring. Biswas, Chakraborty, and Mukherjee (2021) carried out the estimation of stress strength reliability with log-Lindley distribution. Abd El-Monsef, Marei, and Kilany (2022) considered inferences on stress strength reliability of Poisson modified Weibull distribution. Estimation in multi component stress strength model for Pareto distribution based on upper record values was studied in Azhad, Arshad, and Khandelwal (2022). Jose (2022) considered stress strength reliability estimation using discrete phase type distribution.

The RSS (Ranked Set Sampling) concept was first detailed by McIntyre (1952) in an agricultural experiment to estimate the mean of pasture yields. The aim of RSS is to gather observations from the respondents which represents the population than the similar type of observations obtained using SRS (Simple Random Sampling). This technique is used when it is difficult to measure the observations but relatively easier to rank them. Sengupta and Mukhuti (2008) and Muttlak,

Article History
Received : 20 October 2022; Revised : 10 November 2022; Accepted : 22 November 2022; Published : 15 December 2022

To cite this paper

C.J. Rekhana & K. Jayakumar (2022). Estimation of Stress Strength Reliability for New Generalized Pareto Distribution using Advanced Sampling Methods. *Journal of Statistics and Computer Science.* 1(2), 147-158.

Abu-Dayyeh, Saleh, and Al-Sawi (2010) considered the R estimation for exponential distribution under RSS. Hassan, Assar, and Yahya (2015) considered the R estimation under RSS for Burr Type XII distribution. Akgül and Şenoğlu (2017) and Akgül, Acıtaş, and Şenoğlu (2018) also worked on the stress strength reliability estimate based on RSS for Weibull and Lindly distribution respectively. Recently Akgül and Şenoğlu (2022) studied stress strength reliability estimation of Burr Type X distributions based RSS.

The efficiency of RSS method is mainly affected by the possibility of error in the ranking process. Error in ranking increases due to the incorrect measurement of sampling observations. To overcome this drawback of RSS various modified sampling methods have been suggested, see Samawi, Ahmed, and Abu-Dayyeh (1996), Muttlak (1997), Al-Saleh and Al-Kadiri (2000), Muttlak (2003) and Al-Nasser (2007). PRSS (Percentile Ranked Set Sampling) is the modified method of RSS suggested in a study by Muttlak (2003).

Pareto distribution has been used for modeling and analyzing lifetime and income data. Numerous generalizations of Pareto distribution were developed in the recent years in the literature. Jayakumar, Krishnan, and Hamedani (2020) introduced a heavy tailed distribution which signifies the Pareto distribution generalizations. Also this distribution is suitable to be compared with other existing Pareto distribution generalizations. This four parameter distribution is named "New Generalized Pareto distribution" and denoted as $NGP(\alpha,\beta,\gamma,\theta)$.

The probability density function (pdf) of the New Generalized Pareto distribution is

$$f(x;\alpha,\beta,\gamma,\theta)=\frac{\alpha\beta^{\alpha}\theta(1-\gamma)\gamma^{\theta}}{1-\gamma^{\theta}}\frac{x^{\alpha\theta-1}}{(\gamma x^{\alpha}+(1-\gamma)\beta^{\alpha})^{\theta+1}};\quad x>\beta$$

where $\alpha,\beta,\gamma,\theta>0$.

The cumulative distribution function (cdf) is

$$F(x;\alpha,\beta,\gamma,\theta)=\frac{\gamma^{\theta}}{1-\gamma^{\theta}}\left[\frac{x^{\alpha\theta}}{[\gamma x^{\alpha}+(1-\gamma)\beta^{\alpha}]^{\theta}}-1\right];\qquad x>\beta.$$

The hazard rate function is

$$h_F(x;\alpha,\beta,\gamma,\theta)=\frac{\left(\frac{1-\gamma}{\gamma}\right)\frac{\theta\alpha\beta^{\alpha}}{x^{\alpha+1}}}{\left[\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{\beta}{x}\right)^{\alpha}+1\right]\left[1-\gamma^{\theta}\left[\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{\beta}{x}\right)^{\alpha}+1\right]^{\theta}\right]};\quad x>\beta.$$

For NGP distribution, the hazard rate function depicts the decreasing and increasing nature. "The survival function of NGP distribution is a function with regularly varying tails. Also, the distribution belongs to the class $\mathcal{L}$ and is a member of the class of dominated-variation distributions. As a result, the NGP distribution is a member of the class of subexponential distributions, see Jayakumar et al. (2020)". Distributions with regularly varying tails and subexponential distributions have applications in queueing theory, random walk, collective risk theory etc. This makes NGP distribution as a better competitive to other generalizations of Pareto distribution. Consequently the stress strength reliability estimation of NGP distribution is of importance. The objective of the paper is to develop estimation procedure for R under SRS, RSS and PRSS when X and Y were independent $NGP(\alpha_1,1,\gamma_1,\theta_1)$ and $NGP(\alpha_2,1,\gamma_2,\theta_2)$ respectively.

This paper is conducted in sections: ML (Maximum Likelihood) estimation of R based on SRS is detailed in second section. ML estimation of R based on RSS and PRSS are detailed in the third and forth sections. The simulation was carried out in section 5 for comparing performance of R with different sampling techniques. In the last section 6, real data application is presented.

2 ML estimation of R-based on SRS

Assume that $X \sim NGP(\alpha_1, 1, \gamma_1, \theta_1)$; $Y \sim NGP(\alpha_2, 1, \gamma_2, \theta_2)$ are both independent. Then R for NGP distribution is written as,

$$
\begin{aligned}
R &= P(X < Y) \\
&= \int_1^\infty F_X(x) f_Y(x) dx \\
&= \int_1^\infty \frac{\gamma_1^{\theta_1}}{1-\gamma_1^{\theta_1}} \left[\frac{x^{\alpha_1\theta_1}}{[\gamma_1 x^{\alpha_1} + (1-\gamma_1)]^{\theta_1}} - 1 \right] \frac{\alpha_2\theta_2(1-\gamma_2)\gamma_2^{\theta_2}}{1-\gamma_2^{\theta_2}} \frac{x^{\alpha_2\theta_2 - 1}}{(\gamma_2 x^{\alpha_2} + (1-\gamma_2))^{\theta+1}} dx. \qquad (1)
\end{aligned}
$$

Obviously the above integral can not be solved analytically. So we use numerical methods to evaluate R.

Now for obtaining the ML estimate of R, we need to find ML estimate of parameters. Consider two independent SRS samples $NGP(\alpha_1, 1, \gamma_1, \theta_1)$ and $NGP(\alpha_2, 1, \gamma_2, \theta_2)$, such as "$X_i$, $(i = 1, 2, ..., n)$ and Y_j, $(j = 1, 2, ..., m)$" respectively. Then likelihood function based on SRS is written as,

$$
L = \prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} f(y_j)
$$

$$
= \frac{\alpha_1^n \theta_1^n (1-\gamma_1)^n \gamma_1^{\theta_1 n}}{(1-\gamma_1^{\theta_1})^n} \frac{(\prod_{i=1}^n x_i)^{\alpha_1\theta_1 - 1}}{[\prod_{i=1}^n (\gamma_1 x_i^{\alpha_1} + (1-\gamma_1))]^{\theta_1+1}}
\frac{\alpha_2^m \theta_2^m (1-\gamma_2)^m \gamma_2^{\theta_2 m}}{(1-\gamma_2^{\theta_2})^m} \frac{\left(\prod_{j=1}^m y_j\right)^{\alpha_2\theta_2 - 1}}{\left[\prod_{j=1}^m \left(\gamma_2 y_j^{\alpha_2} + (1-\gamma_2)\right)\right]^{\theta_2+1}}.
$$

Further,

$$
\begin{aligned}
\log L =& n\log\alpha_1 + n\log\theta_1 + n\log(1-\gamma_1) + n\theta_1\log\gamma_1 - n\log(1-\gamma_1^{\theta_1}) + \\
& (\alpha_1\theta_1 - 1)\sum_{i=1}^n \log x_i - (\theta_1+1)\sum_{i=1}^n \log\left(\gamma_1 x_i^{\alpha_1} + (1-\gamma_1)\right) + \\
& m\log\alpha_2 + m\log\theta_2 + m\log(1-\gamma_2) + m\theta_2\log\gamma_2 - m\log(1-\gamma_2^{\theta_2}) + \\
& (\alpha_2\theta_2 - 1)\sum_{j=1}^m \log y_j - (\theta_2+1)\sum_{j=1}^m \log\left(\gamma_2 y_j^{\alpha_2} + (1-\gamma_2)\right).
\end{aligned}
$$

For the log - likelihood function, the partial derivatives are given as,

$$
\begin{aligned}
\frac{\partial \log L}{\partial \alpha_1} &= \frac{n}{\alpha_1} + \theta_1 \sum_{i=1}^n \log x_i - (\theta_1+1)\sum_{i=1}^n \frac{\gamma_1 x_i^{\alpha_1}\log x_i}{(\gamma_1 x_i^{\alpha_1} + (1-\gamma_1))}, \\
\frac{\partial \log L}{\partial \gamma_1} &= \frac{-n}{1-\gamma_1} + \frac{n\theta_1}{\gamma_1} + \frac{n\theta_1\gamma_1^{\theta_1-1}}{1-\gamma_1^{\theta_1}} - (\theta_1+1)\sum_{i=1}^n \frac{x_i^{\alpha_1} - 1}{(\gamma_1 x_i^{\alpha_1} + (1-\gamma_1))}, \\
\frac{\partial \log L}{\partial \theta_1} &= \frac{n}{\theta_1} + n\log\gamma_1 + \frac{n\gamma_1^{\theta_1}\log\gamma_1}{1-\gamma_1^{\theta_1}} + \alpha_1\sum_{i=1}^n \log x_i - \sum_{i=1}^n \log(\gamma_1 x_i^{\alpha_1} + (1-\gamma_1)), \\
\frac{\partial \log L}{\partial \alpha_2} &= \frac{m}{\alpha_2} + \theta_2 \sum_{j=1}^m \log y_j - (\theta_2+1)\sum_{j=1}^m \frac{\gamma_2 y_j^{\alpha_2}\log y_j}{(\gamma_2 y_j^{\alpha_2} + (1-\gamma_2))},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \log L}{\partial \gamma_2} &= \frac{-m}{1-\gamma_2} + \frac{m\theta_2}{\gamma_2} + \frac{m\theta_2\gamma_2^{\theta_2-1}}{1-\gamma_2^{\theta_2}} - (\theta_2+1)\sum_{j=1}^m \frac{y_j^{\alpha_2} - 1}{(\gamma_2 y_j^{\alpha_2} + (1-\gamma_2))}, \\
\frac{\partial \log L}{\partial \theta_2} &= \frac{m}{\theta_2} + m\log\gamma_2 + \frac{m\gamma_2^{\theta_2}\log\gamma_2}{1-\gamma_2^{\theta_2}} + \alpha_2\sum_{j=1}^m \log y_j - \sum_{j=1}^m \log(\gamma_2 y_j^{\alpha_2} + (1-\gamma_2)).
\end{aligned}
$$

The ML estimates of the parameters are evaluated by equating the above listed equations with zero and simultaneously solving them. Analytically solving of the equations is not possible. Hence we used $optim()$ function in R software to solve them numerically.

Applying the invariance property of ML estimates, the ML estimate of stress strength reliability R based on SRS, $\hat{R}_{SRS}$ can be attained by using the ML estimates of parameters in Equation (1).

3 ML estimation of R-based on RSS

Let $X_{(i)ik}, (i = 1, 2, ..., m_x); (k = 1, 2, ..., r_x)$ be a ranked set sample observed in $NGP(\alpha_1, 1, \gamma_1, \theta_1)$ with $n = m_x r_x$ sample size, with m_x is set size and number of cycles is given by r_x. Similarly, assume $Y_{(j)jl}, (j = 1, 2, ..., m_y); (l = 1, 2, ..., r_y)$ is the ranked set sample observed from $NGP(\alpha_2, 1, \gamma_2, \theta_2)$ with r_y as the number of cycles and m_y as the set size.
Futrher, based on RSS, the likelihood function is given by,

$$L = \prod_{k=1}^{r_x}\prod_{i=1}^{m_x} f(x_{ik}) \prod_{l=1}^{r_y}\prod_{j=1}^{m_y} f(y_{jl})$$

where,

$$f(x_{ik}) = \frac{m_x!}{(i-1)!(m_x-i)!}\frac{\alpha_1\theta_1(1-\gamma_1)\gamma_1^{i\theta_1}}{(1-\gamma_1{}^{\theta_1})^{m_x}}\frac{x_{ik}^{\alpha_1\theta_1-1}}{(\gamma_1 x_{ik}^{\alpha_1}+(1-\gamma_1))^{\theta_1+1}} \left[\frac{x_{ik}^{\alpha_1\theta_1}}{(\gamma_1 x_{ik}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{i-1}\left[1-\frac{\gamma_1^{\theta_1}x_{ik}^{\alpha_1\theta_1}}{(\gamma_1 x_{ik}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{m_x-i}$$

$$f(x_{jl}) = \frac{m_y!}{(j-1)!(m_y-j)!}\frac{\alpha_2\theta_2(1-\gamma_2)\gamma_2^{j\theta_2}}{(1-\gamma_2{}^{\theta_2})^{m_y}}\frac{y_{jl}^{\alpha_2\theta_2-1}}{(\gamma_2 y_{jl}^{\alpha_2}+(1-\gamma_2))^{\theta_2+1}} \left[\frac{y_{jl}^{\alpha_2\theta_2}}{(\gamma_2 y_{jl}^{\alpha_2}+(1-\gamma_2))^{\theta_2}}\right]^{j-1}\left[1-\frac{\gamma_2^{\theta_2}y_{jl}^{\alpha_2\theta_2}}{(\gamma_2 y_{jl}^{\alpha_2}+(1-\gamma_2))^{\theta_2}}\right]^{m_y-j}.$$

Further, log - likelihood is written as,

$$\begin{aligned}\log L = {} & \log K + n\log\alpha_1 + n\log\theta_1 + n\log(1-\gamma_1) + \theta_1\sum_{k=1}^{r_x}\sum_{i=1}^{m_x} i\log\gamma_1 + \\ & (\alpha_1\theta_1-1)\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}\log x_{ik} - (\theta_1+1)\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}\log(A_{ik}) + \\ & \sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(i-1)\log\left(\frac{x_{ik}^{\alpha_1\theta_1}}{(A_{ik})^{\theta_1}}-1\right) + \sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(m_x-i)\log\left(1-\frac{\gamma_1^{\theta_1}x_{ik}^{\alpha_1\theta_1}}{(A_{ik})^{\theta_1}}\right) + \\ & m\log\alpha_2 + m\log\theta_2 + m\log(1-\gamma_2) + \theta_2\sum_{l=1}^{r_y}\sum_{j=1}^{m_y} j\log\gamma_2 + \\ & (\alpha_2\theta_2-1)\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\log y_{jl} - (\theta_2+1)\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\log(B_{jl}) + \\ & \sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(j-1)\log\left(\frac{y_{jl}^{\alpha_2\theta_2}}{(B_{jl})^{\theta_2}}-1\right) + \sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(m_y-j)\log\left(1-\frac{\gamma_2^{\theta_2}y_{jl}^{\alpha_2\theta_2}}{(B_{jl})^{\theta_2}}\right)\end{aligned}$$

where $K = \prod_{k=1}^{r_x}\prod_{i=1}^{m_x}\frac{m_x!}{(i-1)!(m_x-i)!}\prod_{l=1}^{r_y}\prod_{j=1}^{m_y}\frac{m_y!}{(j-1)!(m_y-j)!}$,

$A_{ik} = (\gamma_1 x_{ik}^{\alpha_1} + (1-\gamma_1))$ and $B_{jl} = (\gamma_2 y_{jl}^{\alpha_2} + (1-\gamma_2))$.

Now for log-likelihood equation,the partial derivatives are given by,

$$\frac{\partial \log L}{\partial \alpha_1} = \frac{n}{\alpha_1} + \theta_1 \sum_{k=1}^{r_x}\sum_{i=1}^{m_x} \log x_{ik} - \gamma_1(\theta_1+1)\sum_{k=1}^{r_x}\sum_{i=1}^{m_x} \frac{x_{ik}^{\alpha_1}\log x_{ik}}{A_{ik}} +$$
$$\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(i-1)\frac{x_{ik}^{\alpha_1\theta_1}\left(A_{ik}\log x_{ik}^{\theta_1} - \gamma_1\theta_1 x_{ik}^{\alpha_1}\log x_{ik}\right)}{A_{ik}(x_{ik}^{\alpha_1\theta_1} - A_{ik}^{\theta_1})} +$$
$$\gamma_1^{\theta_1}\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(m_x - i)\frac{x_{ik}^{\alpha_1\theta_1}\left(A_{ik}\log x_{ik}^{\theta_1} - \gamma_1\theta_1 x_{ik}^{\alpha_1}\log x_{ik}\right)}{A_{ik}\left(\gamma_1^{\theta_1}x_{ik}^{\alpha_1\theta_1} - A_{ik}^{\theta_1}\right)},$$

$$\frac{\partial \log L}{\partial \gamma_1} = \frac{-n}{(1-\gamma_1)} + \sum_{k=1}^{r_x}\sum_{i=1}^{m_x}\frac{i}{\gamma_1} - (\theta+1)\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}\frac{x_{ik}^{\alpha_1}-1}{A_{ik}} -$$
$$\theta_1\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(i-1)\frac{x_{ik}^{\alpha_1\theta_1}(x_{ik}^{\alpha_1}-1)}{A_{ik}(x_{ik}^{\alpha_1\theta_1} - A_{ik}^{\theta_1})} + \theta_1\gamma_1^{\theta_1-1}\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(m_x-i)\frac{x_{ik}^{\alpha_1\theta_1}}{A_{ik}(\gamma_1^{\theta_1}x_{ik}^{\alpha_1\theta_1} - A_{ik}^{\theta_1})},$$

$$\frac{\partial \log L}{\partial \theta_1} = \frac{n}{\theta_1} + \log\gamma_1\sum_{k=1}^{r_x}\sum_{i=1}^{m_x} i + \alpha_1\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}\log x_{ik} - \sum_{k=1}^{r_x}\sum_{i=1}^{m_x}\log A_{ik} +$$
$$\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(i-1)\frac{x_{ik}^{\alpha_1\theta_1}}{x_{ik}^{\alpha_1\theta_1} - A_{ik}^{\theta_1}}\log\left(\frac{x_{ik}^{\alpha_1}}{A_{ik}}\right) +$$
$$\gamma_1^{\theta_1}\sum_{k=1}^{r_x}\sum_{i=1}^{m_x}(m_x-i)\frac{x_{ik}^{\alpha_1\theta_1}}{\gamma_1^{\theta_1}x_{ik}^{\alpha_1\theta_1} - A_{ik}^{\theta_1}}\log\left(\frac{\gamma_1 x_{ik}^{\alpha_1}}{A_{ik}}\right),$$

$$\frac{\partial \log L}{\partial \alpha_2} = \frac{m}{\alpha_2} + \theta_2\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\log y_{jl} - \gamma_2(\theta_2+1)\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\frac{y_{jl}^{\alpha_2}\log y_{jl}}{B_{jl}} +$$
$$\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(j-1)\frac{y_{jl}^{\alpha_2\theta_2}\left(B_{jl}\log y_{jl}^{\theta_2} - \gamma_2\theta_2 y_{jl}^{\alpha_2}\log y_{jl}\right)}{B_{jl}(y_{jl}^{\alpha_2\theta_2} - B_{jl}^{\theta_2})} +$$
$$\gamma_2^{\theta_2}\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(m_y-j)\frac{y_{jl}^{\alpha_2\theta_2}\left(B_{jl}\log y_{jl}^{\theta_2} - \gamma_2\theta_2 y_{jl}^{\alpha_2}\log y_{jl}\right)}{B_{jl}\left(\gamma_2^{\theta_2}y_{jl}^{\alpha_2\theta_2} - B_{jl}^{\theta_2}\right)},$$

$$\frac{\partial \log L}{\partial \gamma_2} = \frac{-m}{(1-\gamma_2)} + \sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\frac{j}{\gamma_2} - (\theta+2)\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\frac{y_{jl}^{\alpha_2}-1}{B_{jl}} -$$
$$\theta_2\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(j-1)\frac{y_{jl}^{\alpha_2\theta_2}(y_{jl}^{\alpha_2}-1)}{B_{jl}(y_{jl}^{\alpha_2\theta_2} - B_{jl}^{\theta_2})} + \theta_2\gamma_2^{\theta_2-1}\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(m_y-j)\frac{y_{jl}^{\alpha_2\theta_2}}{B_{jl}(\gamma_2^{\theta_2}Y_{jl}^{\alpha_2\theta_2} - B_{jl}^{\theta_2})},$$

$$\frac{\partial \log L}{\partial \theta_2} = \frac{m}{\theta_2} + \log\gamma_2\sum_{l=1}^{r_y}\sum_{j=1}^{m_y} j + \alpha_2\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\log y_{jl} - \sum_{l=1}^{r_y}\sum_{j=1}^{m_y}\log B_{jl} +$$
$$\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(j-1)\frac{y_{jl}^{\alpha_2\theta_2}}{y_{jl}^{\alpha_2\theta_2} - B_{jl}^{\theta_2}}\log\left(\frac{y_{jl}^{\alpha_2}}{B_{jl}}\right) +$$
$$\gamma_2^{\theta_2}\sum_{l=1}^{r_y}\sum_{j=1}^{m_y}(m_y-j)\frac{y_{jl}^{\alpha_2\theta_2}}{\gamma_2^{\theta_2}y_{jl}^{\alpha_2\theta_2} - B_{jl}^{\theta_2}}\log\left(\frac{\gamma_2 y_{jl}^{\alpha_2}}{B_{jl}}\right).$$

ML estimates of unknown parameters under RSS are calculated by equating above equations to zero and solving simultaneously. But it is difficult so solve these equations analytically, so similar to estimation of parameters in SRS, we use $optim()$ function in R software. Then using the invariance property of MLE, ML estimate of R using RSS, namely $\hat{R}_{RSS}$, is calculated by substituting ML estimates of the parameters in Equation (1).

4 ML estimation of R-based on PRSS

The following section deals with the ML estimation of stress strength reliability R using PRSS. Here the inference procedure are considered for even and odd set sizes individually.

Case 1: Odd set sizes: Assume that a_x, b_x, a_y and b_y are nearest integer value of $p[m_x+1], q[m_x+1], p[m_y+1]$ and $q[m_y+1]$, with $0<p<1$ and $q=1-p$. Also ϑ and ω are given by $\frac{m_x+1}{2}$ and $\frac{m_y+1}{2}$.

Assume $\{X_{(a_x)ik},\ i=1,2,...,\vartheta-1;\ k=1,2,...,r_x\} \bigcup \{X_{(\vartheta)ik},\ i=\vartheta;\ k=1,2,...,r_x\} \bigcup \{X_{(b_x)ik},\ i=\vartheta+1,...,m_x;\ k=1,2,...,r_x\}$ be the percentile ranked set sample drawn from $NGP(\alpha_1,1,\gamma_1,\theta_1)$ with sample size $n=m_x r_x$, where m_x and r_x are the size of the set and total cycles respectively. Similarly let $\{Y_{(a_y)jl},\ j=1,2,...,\omega-1;\ l=1,2,...,r_y\} \bigcup \{Y_{(\omega)jl},\ j=\omega;\ l=1,2,...,r_y\} \bigcup \{Y_{(b_y)jl},\ j=\omega+1,...,m_y;\ l=1,2,...,r_y\}$ be percentile ranked set sample drawn from $NGP(\alpha_2,1,\gamma_2,\theta_2)$ with sample size $m=m_y r_y$, where m_y and r_y defines size of the set and total cycles respectively. Then, based on PRSS, likelihood function is,

$$L=\prod_{k=1}^{r_x}\prod_{i=1}^{\vartheta-1} f(x_{(a_x)ik})\prod_{k=1}^{r_x} f(x_{(\vartheta)\vartheta k})\prod_{k=1}^{r_x}\prod_{i=\vartheta+1}^{m_x} f(x_{(b_x)ik})$$
$$\prod_{l=1}^{r_y}\prod_{j=1}^{\omega-1} f(y_{(a_y)jl})\prod_{l=1}^{r_y} f(y_{(\omega)\omega l})\prod_{l=1}^{r_y}\prod_{j=\omega+1}^{m_y} f(y_{(b_y)jl})$$

where

$$f(x_{(a_x)})=\frac{m_x!}{(a_x-1)!(m_x-a_x)!}\frac{\alpha_1\theta_1(1-\gamma_1)\gamma_1^{a_x\theta_1}}{(1-\gamma_1{}^{\theta_1})^{m_x}}\frac{x_{a_x}^{\alpha_1\theta_1-1}}{(\gamma_1 x_{a_x}^{\alpha_1}+(1-\gamma_1))^{\theta_1+1}}$$
$$\left[\frac{x_{a_x}^{\alpha_1\theta_1}}{(\gamma_1 x_{a_x}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{a_x-1}\left[1-\frac{\gamma_1^{\theta_1}x_{a_x}^{\alpha_1\theta_1}}{(\gamma_1 x_{a_x}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{m_x-a_x}$$
$$f(x_{(b_x)})=\frac{m_x!}{(b_x-1)!(m_x-b_x)!}\frac{\alpha_1\theta_1(1-\gamma_1)\gamma_1^{b_x\theta_1}}{(1-\gamma_1{}^{\theta_1})^{m_x}}\frac{x_{b_x}^{\alpha_1\theta_1-1}}{(\gamma_1 x_{b_x}^{\alpha_1}+(1-\gamma_1))^{\theta_1+1}}$$
$$\left[\frac{x_{b_x}^{\alpha_1\theta_1}}{(\gamma_1 x_{b_x}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{b_x-1}\left[1-\frac{\gamma_1^{\theta_1}x_{b_x}^{\alpha_1\theta_1}}{(\gamma_1 x_{b_x}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{m_x-b_x}$$
$$f(x_{(\vartheta)})=\frac{m_x!}{(\vartheta-1)!(m_x-\vartheta)!}\frac{\alpha_1\theta_1(1-\gamma_1)\gamma_1^{\vartheta\theta_1}}{(1-\gamma_1{}^{\theta_1})^{m_x}}\frac{x_{\vartheta}^{\alpha_1\theta_1-1}}{(\gamma_1 x_{\vartheta}^{\alpha_1}+(1-\gamma_1))^{\theta_1+1}}$$
$$\left[\frac{x_{\vartheta}^{\alpha_1\theta_1}}{(\gamma_1 x_{\vartheta}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{\vartheta-1}\left[1-\frac{\gamma_1^{\theta_1}x_{\vartheta}^{\alpha_1\theta_1}}{(\gamma_1 x_{\vartheta}^{\alpha_1}+(1-\gamma_1))^{\theta_1}}\right]^{m_x-\vartheta}$$

In a similar manner we can define $f(y_{(a_y)}), f(y_{(b_y)})$ and $f(y_{(\omega)})$.

Case 2: Even set sizes: Here, reliability estimation is carried out when the independent random variables X and Y are drawn from NGP based on PRSS with even set size.

Let $\{X_{(a_x)ik},\ i=1,2,...,\frac{m_x}{2};\ k=1,2,...,r_x\} \bigcup \{X_{(b_x)ik},\ i=\frac{m_x}{2}+1,...,m_x;\ k=1,2,...,r_x\}$ and $\{Y_{(a_y)jl},\ j=1,2,...,\frac{m_y}{2};\ l=1,2,...,r_y\} \bigcup \{Y_{(b_y)jl},\ j=\frac{m_y}{2}+1,...,m_y;\ l=1,2,...,r_y\}$ be percentile ranked set samples drawn from $NGP(\alpha_1,1,\gamma_1,\theta_1)$ and $NGP(\alpha_2,1,\gamma_2,\theta_2)$ with even set sizes.

Hence likelihood function is,

$$L = \prod_{k=1}^{r_x} \prod_{i=1}^{\frac{m_x}{2}} f(x_{(a_x)ik}) \prod_{k=1}^{r_x} \prod_{\frac{m_x}{2}+1}^{m_x} f(x_{(b_x)ik})$$
$$\prod_{l=1}^{r_y} \prod_{j=1}^{\frac{m_y}{2}} f(y_{(a_y)jl}) \prod_{l=1}^{r_y} \prod_{j=\frac{m_y}{2}+1}^{m_y} f(y_{(b_y)jl}).$$

For finding the ML estimate of the parameters based on PRSS for even and odd set sizes, partial derivatives of the log-likelihood equation is equated to zero, and solve them simultaneously using *optim*() function in R software. Hence applying the invariance property of ML estimates, ML estimate of R based on PRSS, namely $\hat{R}_{PRSS}$, is obtained by substituting ML estimates of the parameters in Equation (1).

5 Simulation study

Next we conduct simulation to compare the estimators performance of stress strength reliability using diffrent sampling methods. Relative efficiency(RE) and Mean square error(MSE) measures are considered for comparing the reliability estimates . In this simulation, true values of the parameters were chosen as, $(\alpha_1, \gamma_1, \theta_1, \alpha_2, \gamma_2, \theta_2) = (1,2,1,2,2,1), (2,2,1,2,2,1), (1,3,3,1,3,.2)$ and $(1,2,1.5,2,2,.5)$ giving $R = .3531, .4067, .5$ and $.7192$. Also $(m_x, m_y) = (2,2), (2,4), (4,4), (4,6), (6,6)$, $(6,8)$ and $r_x = r_y = 10$ for both RSS and PRSS. $(n, m) = (20,20), (20,40), (40,40), (40,60), (60,60)$ and $(60,80)$ are the sample sizes selected for SRS.

We generate 10000 samples from $NGP(\alpha_1, 1, \gamma_1, \theta_1)$ and $NGP(\alpha_2, 1, \gamma_2, \theta_2)$ and calculate stress strength reliability R based on SRS, RSS and PRSS. The results were compared using MSE and RE. The definition of MSE and RE are as follows,

$$RE_1 = \frac{MSE(\hat{R}_{SRS})}{MSE(\hat{R}_{RSS})}, \quad RE_2 = \frac{MSE(\hat{R}_{SRS})}{MSE(\hat{R}_{PRSS})} \text{ and } RE_3 = \frac{MSE(\hat{R}_{RSS})}{MSE(\hat{R}_{PRSS})}.$$

where $MSE(\hat{R}) = E(\hat{R} - R)^2$. The Table 1 below lists the results.

- From Table 1, according to RE_1 values, we can see that the estimate of stress strength reliability R under RSS performs better than the corresponding ones under SRS in all cases.
- According to RE_2 values, the estimate of stress strength reliability R under PRSS performs better as compared to its corresponding ones based on SRS in all cases.
- In maximum of the cases the estimate of stress strength reliability R under PRSS performs better as compared to its corresponding ones based on RSS according to RE_3.
- Also as the set size (m_x, m_y) increases, MSE of estimates of R under PRSS, RSS and SRS decreases. Also RE_1 and RE_2 values increases when the set sizes (m_x, m_y) increases.

6 Real life data application

In the following section, real data set is analyzed for illustrating the use of proposed methodology. The data sets used for analysis is the data of fibre strength is listed in Crowder (2000). Random samples of 168 failure stress observed in hybrid bundles of the parallel carbon fibres. From this we select two data sets which gives the breaking strengths of single carbon fibres of varying lengths. Data sets are given in Table 2 and Table 3.

Further with TTT plot (Total Time on Test), the shape of hazard rate function of the data can be identified.

Table 1: *MSE and RE of $\hat{R}$ based on SRS, RSS and PRSS.*

R	(m_x, m_y)	(n, m)	MSE(SRS)	MSE(RSS)	MSE(PRSS)	RE_1	RE_2	RE_3
.3531	(2,2)	(20,20)	0.0081	0.0054	0.0053	1.5081	1.5356	1.0182
	(2,4)	(20,40)	0.0054	0.0035	0.0034	1.5676	1.5996	1.0204
	(4,4)	(40,40)	0.0028	0.0015	0.0015	1.8622	1.8680	1.0031
	(4,6)	(40,60)	0.0022	0.0011	0.0011	1.8575	1.8688	1.0060
	(6,6)	(60,60)	0.0020	0.0009	0.00092	2.1478	2.1930	1.0210
	(6,8)	(60,80)	0.0018	0.0008	0.00078	2.2336	2.3069	1.0327
.4067	(2,2)	(20,20)	0.0086	0.0061	0.0056	1.4358	1.5452	1.0761
	(2,4)	(20,40)	0.0074	0.0051	0.0043	1.4644	1.6943	1.1569
	(4,4)	(40,40)	0.0024	0.0015	0.0013	1.6245	1.7671	1.0877
	(4,6)	(40,60)	0.0024	0.0012	0.0013	1.9087	1.8965	0.9935
	(6,6)	(60,60)	0.0014	0.0007	0.0006	1.9091	2.1749	1.1391
	(6,8)	(60,80)	0.0013	0.0007	0.0006	1.9323	2.2132	1.1453
.5	(2,2)	(20,20)	0.0097	0.0062	0.0062	1.5452	1.566	1.0136
	(2,4)	(20,40)	0.0079	0.0048	0.0047	1.6495	1.6795	1.0182
	(4,4)	(40,40)	0.0031	0.0019	0.0018	1.6443	1.6831	1.0236
	(4,6)	(40,60)	0.0027	0.0015	0.0014	1.7818	1.9318	1.0841
	(6,6)	(60,60)	0.0021	0.0010	0.0009	1.9724	2.1634	1.0968
	(6,8)	(60,80)	0.0019	0.0009	0.0008	2.1809	2.2353	1.0249
.7192	(2,2)	(20,20)	0.0059	0.0050	0.0046	1.1769	1.27434	1.0827
	(2,4)	(20,40)	0.0036	0.0029	0.0025	1.2455	1.4295	1.1477
	(4,4)	(40,40)	0.0018	0.0013	0.0011	1.3698	1.5996	1.1677
	(4,6)	(40,60)	0.0019	0.0012	0.0010	1.5040	1.8249	1.2133
	(6,6)	(60,60)	0.0014	0.0008	0.0007	1.6192	1.8317	1.1312
	(6,8)	(60,80)	0.0013	0.0007	0.0006	1.8231	2.0870	1.1447

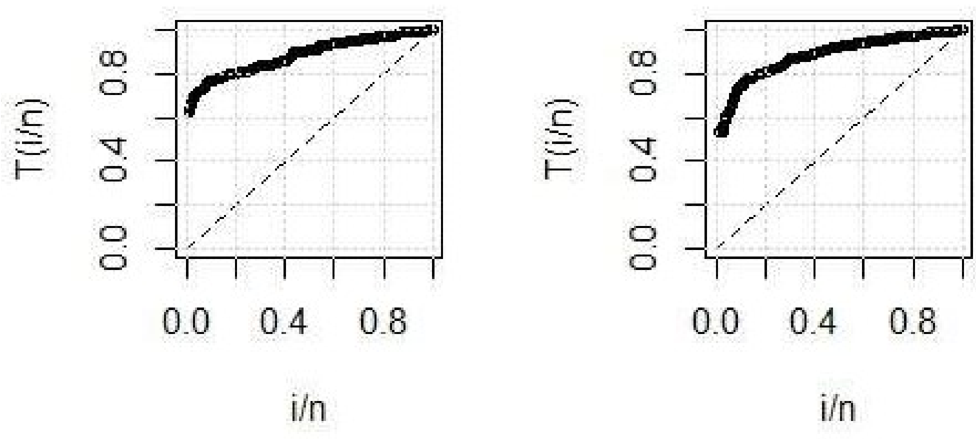

Figure 1: *The scaled TTT plot of data X and data Y respectively*

As per Figure 1, it can be observed that, the hazard rate function of both data sets are increasing. As mentioned in section 1, NGP distribution is used for modelling data sets having increasing

Table 2: *Data X: Breaking strengths of single carbon fibres of length 10*

1.901	2.132	2.203	2.228	2.257	2.350	2.361	2.396
2.397	2.445	2.454	2.454	2.474	2.518	2.522	2.525
2.532	2.575	2.614	2.616	2.618	2.624	2.659	2.675
2.738	2.740	2.856	2.917	2.928	2.937	2.937	2.977
2.996	3.030	3.125	3.139	3.145	3.220	3.223	3.235
3.243	3.264	3.272	3.294	3.332	3.346	3.377	3.408
3.435	3.493	3.501	3.537	3.554	3.562	3.628	3.852
3.871	3.886	3.971	4.024	4.027	4.225	4.395	5.020

Table 3: *Data Y: Breaking strengths of single carbon fibres of length 20*

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865
1.944	1.958	1.966	1.997	2.006	2.021	2.027	2.055
2.063	2.098	2.140	2.179	2.224	2.240	2.253	2.270
2.272	2.274	2.301	2.301	2.339	2.359	2.382	2.382
2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535
2.554	2.566	2.570	2.586	2.629	2.633	2.642	2.648
2.684	2.697	2.726	2.770	2.773	2.800	2.809	2.818
2.821	2.848	2.880	2.954	3.012	3.067	3.084	3.090
3.096	3.128	3.233	3.433	3.585	3.585		

failure rate character. So we fit NGP distribution for both data sets separately. For fitting, we first calculate the distribution parameters. Further, we also need to analyze the goodness of fit of the NGP distribution for the data. In this case, $-\log L$ and KS statistic (Kolmogorov Smirnov) is used along with p-value. Estimated parameters, $-\log L$, KS, p value for both the data sets are presented in Table 4.

Table 4: *Estimates of the parameters, -log L, KS and p values for data sets.*

Data Set	Sample Size	α	γ	θ	$-\log L$	K-S	p value
X	64	7.0033	0.0014	2.3737	58.0598	0.0899	0.6789
Y	70	11.0985	0.00002	0.5330	49.3069	0.0419	0.9997

From Table 4, Figure 2 and Figure 3, we can see that NGP distribution for both the data sets fits well. So we are choosing these data sets to select samples based on PRSS, RSS and SRS. For selecting samples via SRS we take the size of the samples for X and Y as $n = 16, m = 20$. In case of RSS and PRSS, we take $m_x = 4$ and $r_x = 4$ for data X and $m_y = 2$ and $r_y = 10$ for data Y. Also for $n = 64$ and $m = 70$ observations, R is estimated to be 0.2277. The MSEs and means of R estimates based on 10,000 replications of every sampling method is listed in Table 5.

Table 5: *SRS, RSS and PRSS based ML estimates of R.*

	$\hat{R}_{SRS}$	$\hat{R}_{RSS}$	$\hat{R}_{PRSS}$
Mean	0.2285	0.2289	0.2266
Bias	0.0008	0.0012	-0.0011
MSE	0.0044	0.0029	0.0026

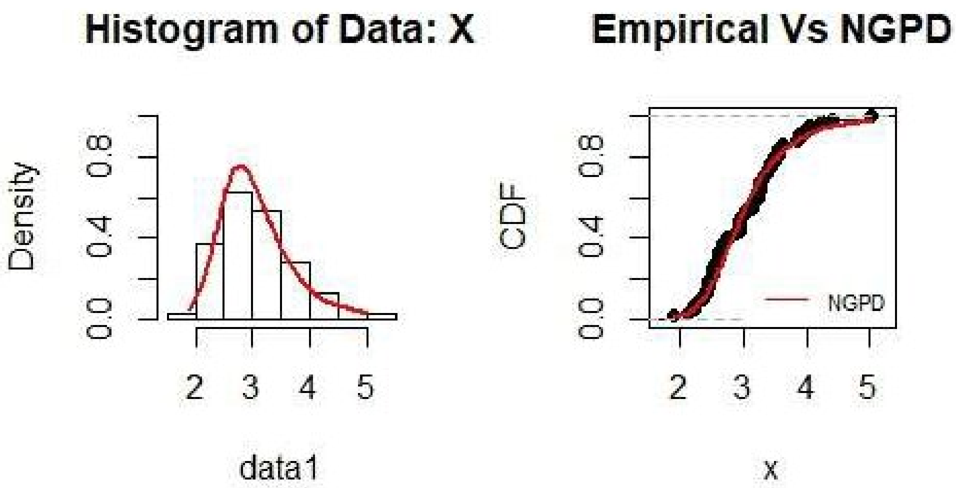

Figure 2: *Fitted pdf histogram and fitted cdf with empirical cdf for data: X.*

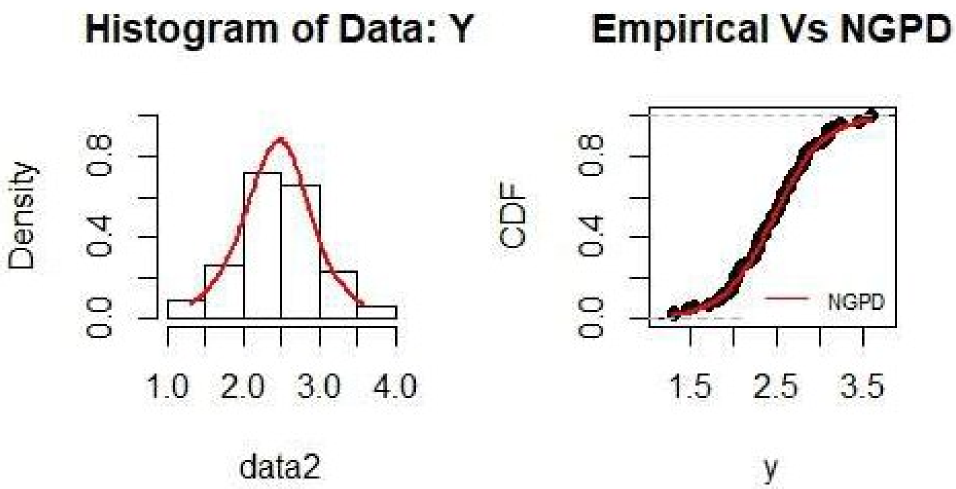

Figure 3: *Fitted pdf histogram and fitted cdf with empirical cdf for data: Y.*

According to Table 5 it can be said that the estimated values of R based on $n = 16$ and $m = 20$ sampling units using different sampling designs are closer to estimated value of R calculated using the complete data set. In terms of MSEs, it can be said that $\hat{R}_{PRSS}$ perform better as compared to $\hat{R}_{SRS}$ and $\hat{R}_{RSS}$.

7 Conclusion

In this paper, the ML estimates of the stress strength reliability R based on SRS, RSS and PRSS are obtained, when the stress and strength are independently following NGP distribution. The performance of the proposed estimators are compared using a Monte Carlo simulation study. From the simulation study it is clear that PRSS performs better than RSS and SRS. Also we can see that, the efficiency of all estimates increases as the set size increases. The results from the simulation study is supported by a real life data set.

Acknowledgement

The authors are thankful to the the Editor-in-Chief and anonymous reviewers for their valuable suggestions and comments. The first author wishes to thank the University Grants Commission, New Delhi, India for the financial assistance in the form of Junior Research Fellowship.

References

Abd El-Monsef, M. M. E., Marei, G. A., & Kilany, N. M. (2022). Poisson modified Weibull distribution with inferences on stress-strength reliability model. *Quality & Reliability Engineering International*, *38*(5), 2649–2669.

Akgül, F. G., Acıtaş, Ş., & Şenoğlu, B. (2018). Inferences on stress–strength reliability based on ranked set sampling data in case of Lindley distribution. *Journal of Statistical Computation & Simulation*, *88*(15), 3018–3032.

Akgül, F. G., & Şenoğlu, B. (2017). Estimation of P(X< Y) using ranked set sampling for the Weibull distribution. *Quality Technology & Quantitative Management*, *14*(3), 296–309.

Akgül, F. G., & Şenoğlu, B. (2022). Inferences for stress–strength reliability of Burr Type X distributions based on ranked set sampling. *Communications in Statistics-Simulation & Computation*, *51*(6), 3324–3340.

Al-Mutairi, D., Ghitany, M., & Kundu, D. (2013). Inferences on stress-strength reliability from Lindley distributions. *Communications in Statistics-Theory & Methods*, *42*(8), 1443–1463.

Al-Nasser, A. D. (2007). L ranked set sampling: A generalization procedure for robust visual sampling. *Communications in Statistics- Simulation & Computation*, *36*(1), 33–43.

Al-Omari, A. I., Almanjahie, I. M., Hassan, A. S., & Nagy, H. F. (2020). Estimation of the stress-strength reliability for exponentiated Pareto distribution using median and ranked set sampling methods. *CMC-Computers, Materials & Continua*, *64*(2), 835–857.

Al-Saleh, M. F., & Al-Kadiri, M. A. (2000). Double-ranked set sampling. *Statistics & Probability Letters*, *48*(2), 205–212.

Azhad, Q. J., Arshad, M., & Khandelwal, N. (2022). Statistical inference of reliability in multi component stress strength model for Pareto distribution based on upper record values. *International Journal of Modelling & Simulation*, *42*(2), 319–334.

Birnbaum, Z. (1956). On a use of the Mann-Whitney statistic. In *Proceedings of the third Berkeley symposium on Mathematical Statistics & Probability* (Vol. 1, pp. 13–17).

Biswas, A., Chakraborty, S., & Mukherjee, M. (2021). On estimation of stress–strength reliability with log-Lindley distribution. *Journal of Statistical Computation & Simulation*, *91*(1), 128–150.

Church, J. D., & Harris, B. (1970). The estimation of reliability from stress-strength relationships. *Technometrics*, *12*(1), 49–54.

Crowder, M. (2000). Tests for a family of survival models based on extremes. In *Recent Advances in Reliability Theory* (pp. 307–321). Springer.

Ghitany, M., Al-Mutairi, D. K., & Aboukhamseen, S. (2015). Estimation of the reliability of a stress-strength system from power Lindley distributions. *Communications in Statistics-Simulation & Computation*, *44*(1), 118–136.

Hassan, A., Assar, S., & Yahya, M. (2015). Estimation of P(Y<X) for Burr distribution under several modifications for ranked set sampling. *Australian Journal of Basic and Applied Sciences*, *9*(1), 124–140.

Jayakumar, K., Krishnan, B., & Hamedani, G. G. (2020). On a new generalization of Pareto distribution and its applications. *Communications in Statistics-Simulation & Computation*, *49*(5), 1264–1284.

Johnson, R. A. (1988). 3 stress-strength models for reliability. *Handbook of Statistics*, *7*, 27–54.

Jose, J. K. (2022). Estimation of stress-strength reliability using discrete Phase type distribution. *Communications in Statistics-Theory & Methods*, *51*(2), 368–386.

Kotz, S., & Pensky, M. (2003). *The stress-strength model and its generalizations: theory and applications.* World Scientific.

Krishnamoorthy, K., Mukherjee, S., & Guo, H. (2007). Inference on reliability in two-parameter exponential stress–strength model. *Metrika*, *65*(3), 261–273.

Kundu, D., & Gupta, R. D. (2005). Estimation of P[Y< X] for generalized exponential distribution. *Metrika*, *61*(3), 291–308.

McIntyre, G. (1952). A method for unbiased selective sampling, using ranked sets. *Australian Journal of Agricultural Research*, *3*(4), 385–390.

Muttlak, H. A. (1997). Median ranked set sampling. *Journal of Applied Statistical Science*, *6*(4), 245–255.

Muttlak, H. A. (2003). Modified ranked set sampling methods. *Pakistan Journal of Statistics*, *19*(3), 315–324.

Muttlak, H. A., Abu-Dayyeh, W., Saleh, M., & Al-Sawi, E. (2010). Estimating P(Y< X) using ranked set sampling in case of the exponential distribution. *Communications in Statistics-Theory & Methods*, *39*(10), 1855–1868.

Raqab, M. Z., Madi, M. T., & Kundu, D. (2008). Estimation of P(Y< X) for the three-parameter generalized exponential distribution. *Communications in Statistics-Theory & Methods*, *37*(18), 2854–2864.

Rezaei, A., Sharafi, M., Behboodian, J., & Zamani, A. (2018). Inferences on stress–strength parameter based on GLD5 distribution. *Communications in Statistics-Simulation & Computation*, *47*(5), 1251–1263.

Saini, S., Chaturvedi, A., & Garg, R. (2021). Estimation of stress–strength reliability for generalized Maxwell failure distribution under progressive first failure censoring. *Journal of Statistical Computation & Simulation*, *91*(7), 1366–1393.

Samawi, H. M., Ahmed, M. S., & Abu-Dayyeh, W. (1996). Estimating the population mean using extreme ranked set sampling. *Biometrical Journal*, *38*(5), 577–586.

Sengupta, S., & Mukhuti, S. (2008). Unbiased estimation of P(X> Y) using ranked set sample data. *Statistics*, *42*(3), 223–230.